Characteristics and Simulated Performance of Short Convolutional Codes: Length 7, Rate 1/3

J. W. Layland
Communications Systems Research Section

This article compares the characteristics and performance of two near-optimal constraint-length 7, rate 1/3 convolutional codes. Performance estimates are based upon software simulations for the additive white Gaussian noise channel.

I. Introduction

It is currently expected that in the near future the DSN will be asked to support a series of missions utilizing short-constraint-length convolutional codes. A maximum-likelihood or Viterbi-algorithm decoder will be utilized in the tracking stations. The first application will be the Mariner Jupiter/Saturn 1977 (MJS'77) mission for which a constraint-length 7, rate ½ code has been selected as the baseline design. That particular rate ½ code is generally accepted as being optimal and has been used elsewhere in currently available hardware (Refs. 1, 2). No specific commitment to a rate ⅓ code exists at this time, but it has been shown that if sufficient bandwidth exists, the rate ⅓ codes generally require approximately 0.4 dB less signal-to-noise ratio (SNR) for equivalent performance than do the rate ½ codes of the same constraint-length (Ref. 3).

Since additionally neither the encoder nor the decoder for a rate ⅓ code is significantly more complex than that for a rate ½ code of the same constraint length, there is considerable motivation for deploying within the DSN a decoder with the multi-mission capability for decoding both rate ⅓ and rate ½ codes of the chosen length (i.e., 7). It is the express purpose of the work described in this article to assist in the selection of the best length 7, rate ⅓ convolutional code. Two codes are discussed in detail here. Of all the 7:⅓ codes which conceptually exist, most are very poor performers, and all are believed to be poorer performers than these two codes.

II. A Good 7:1/3 Code With Weight 14

The first code to be discussed has a generator matrix which may be represented in octal as 7566127. This matrix has weight 14 (14 one's), and hence the code has a maximum free distance of 14. The optimum 7:½ code is embedded within this code. This code has been widely distributed, having been identified by Odenwalder (Ref. 2) as the optimum 7:⅓ code, and later included in LINKABIT (Ref. 4) and Shuttle (Ref. 5) reports. Table 1 lists the lowest weight code words for this code. No code words of odd weight exist. The code is non-transparent so that the 180-deg subcarrier's phase ambiguity must be resolved by the decoder as it is acquiring node-synchronization and before it begins decoding.

III. A Good 7:1/3 Code With Weight 15

The second code of interest has a generator matrix which may be represented in octal as 7576127. The generator matrix of this code is separated by one bit from the generator matrix of the previous code, and it also contains the optimum 7:½ code embedded within it. We were motived to construct this code from the previous one by noting that the upper bound to achievable free-distance is 15 (Ref. 6), and hence that a better code might be available. Table 2 lists the lowest-weight code words for this code. The free-distance bound of 15 is achieved. Hence, at extremely low error probability, where the code performance is almost entirely defined by the free-distance and the code-words at the free distance, this code will have somewhat lower error probability than the weight-14 code. This code is transparent so the 180 deg subcarrier phase ambiguity is not detected by the decoder, but is passed through to the data user.

IV. Comparison by Simulation

Performance of maximum-likelihood decoding of both 7:⅓ codes has been simulated for the additive-white Gaussian Noise Channel. The simulation used 4-bit (16-level) quantization of the input symbols, and a decoder path memory of 64-bits. Decoder bits were taken from the most likely path. The software decoder operates at a relatively unimpressive 6×10^5 bits per hour, thereby limiting practical sample-sizes to about 4×10^6 bits.

Figure 1 shows the simulated bit error probability for the two codes. Data points at 1.6 dB and above represent 4×10^6 bits. The two data points at 1.1 and 1.4 dB represent only 4×10^5 bits. At each value of Eb/N_0 simulated, an identical noise sequence was input to both coders. The noise sequences were distinct for distinct values of Eb/N_0. Because errors in the output of the decoder occur in bursts, rather than independently, confidence intervals for this simulated error probability must be assigned according to the number of bursts, or error events, which occur within the simulation run. The one-sigma confidence intervals were computed on this basis and are listed in Table 3. They are distinct only at 2.4 dB, and they overlap to an ever-increasing extent as Eb/N_0 is lowered.

A three-sigma confidence interval for either code would in all cases include the data point for the other code. Thus while the simulation implies that the weight 15 code performs better by a miniscule 0.02 dB at 10^{-3} bit error rate, it does not provide a statistically significant conclusion. Increasing the sample-size by a factor of 10 or more, as would be needed to achieve a statistically significant differentiation of the two codes, does not seem feasible without a much faster decoder.

Most data transmitted from a spacecraft does not consist of independent bits, but consists instead of instrument data words, each of several bits in duration; e.g., "pixels," or picture elements which have a nominal 8-bit length. As a result, the data user is often interested more in the error clustering characteristics of the coded channel, than in the bit error probability *per se.* Figure 2 shows this clustering characteristic in two forms: the first is the probability that an error burst occurs (typical bursts would be 3-15 bits), and the second is the probability that an 8-bit pixel contains an error. As before, there is no statistically significant difference between the two codes.

V. Conclusion

The comparison between the two convolutional codes with constraint length 7 and rate ⅓ discussed here has shown no significant differences in error performance for bit error probabilities in the neighborhood of 10^{-3}. As a result, the choice between them should be based upon the operational implications of code transparency and the subcarrier phase ambiguity.

References

1. Linkabit Corporation: Sales and Advertising Literature, San Diego, Calif., 1973 and 1974.
2. Odenwalder, *Optimal Decoding of Convolutional Codes*, Ph.D. Thesis, Systems Science Department, University of California, Los Angeles, 1970.
3. Layland, J. W., "Information Systems: Performance of Short Constraint Length Convolutional Codes, and a Heuristic Code-Construction Algorithm," in *The Deep Space Network*, Space Programs Summary 37-64, Vol. II, pp. 41 to 44, Jet Propulsion Laboratory, Pasadena, Calif., Aug. 31, 1970.
4. Gilhousen, K. S., et al., *Coding Systems Study for High Data Rate Telemetry Link*, Linkabit Corp., San Diego, Calif., January 1971.
5. *Shuttle Orbiter/GSFC Communications and Tracking Interface Control Document (Preliminary) ICD-2-0D044.* Lyndon B. Johnson Space Center, Houston, Texas, January 1974.
6. McEliece, R. J., and Layland, J. W., "An Upper Bound to the Free Distance of a Tree Code," in *Supporting Research and Advanced Development*, Space Programs Summary 37-62, Vol. III, pp. 62–64, Jet Propulsion Laboratory, Pasadena, Calif., April 30, 1970.

Table 1. Low-weight code words of 7566127

Weight	Number of 1's at this weight	Code words (information bits)
14	1	1
15	None	
16	2 (20 Total)	1001
	3	10101
	3	1011
	2	11
	3	11001
	3	111
	4	1111
17	None	
18	2 (53 Total)	101
	4	10110001
	3	110001
	5	110010101
	5	1100111
	4	111001
	4	11101
	7	11110111
	8	1111011101
	5	11111
	6	111111
19	None	
20	184 Total	35 distinct sequences
21	None	
22	555 Total	90 distinct sequences

Table 2. Low-weight code words of 7576127

Weight	Number of 1's at this weight	Code words (information bits)
14	None	
15	1 (7 Total)	1
	3	11001
	3	111
16	2 (8 Total)	101
	2	111
	4	111001
17	3 (22 Total)	10101
	3	1011
	5	101111
	3	110001
	5	1100111
	3	1101
18	2 (44 Total)	1001
	4	10110001
	6	1100111001
	6	11001111
	8	1100111111
	4	11011
	4	1110001
	4	1111
	6	111111
19	22 Total	4 distinct sequences
20	94 Total	18 distinct sequences
21	222 Total	35 distinct sequences
22	282 Total	45 distinct sequences

Table 3. One-sigma confidence ranges for estimated bit error probability

Eb/N_0	Code 7566127		Code 7576127	
	Low estimate	High estimate	Low estimate	High estimate
1.6	5.75×10^{-3}	5.95×10^{-3}	5.72×10^{-3}	5.92×10^{-3}
1.8	3.78×10^{-3}	3.94×10^{-3}	3.70×10^{-3}	3.86×10^{-3}
2.0	2.48×10^{-3}	2.58×10^{-3}	2.37×10^{-3}	2.49×10^{-3}
2.2	1.50×10^{-3}	1.58×10^{-3}	1.44×10^{-3}	1.52×10^{-3}
2.4	8.90×10^{-4}	9.55×10^{-4}	8.21×10^{-4}	8.86×10^{-4}

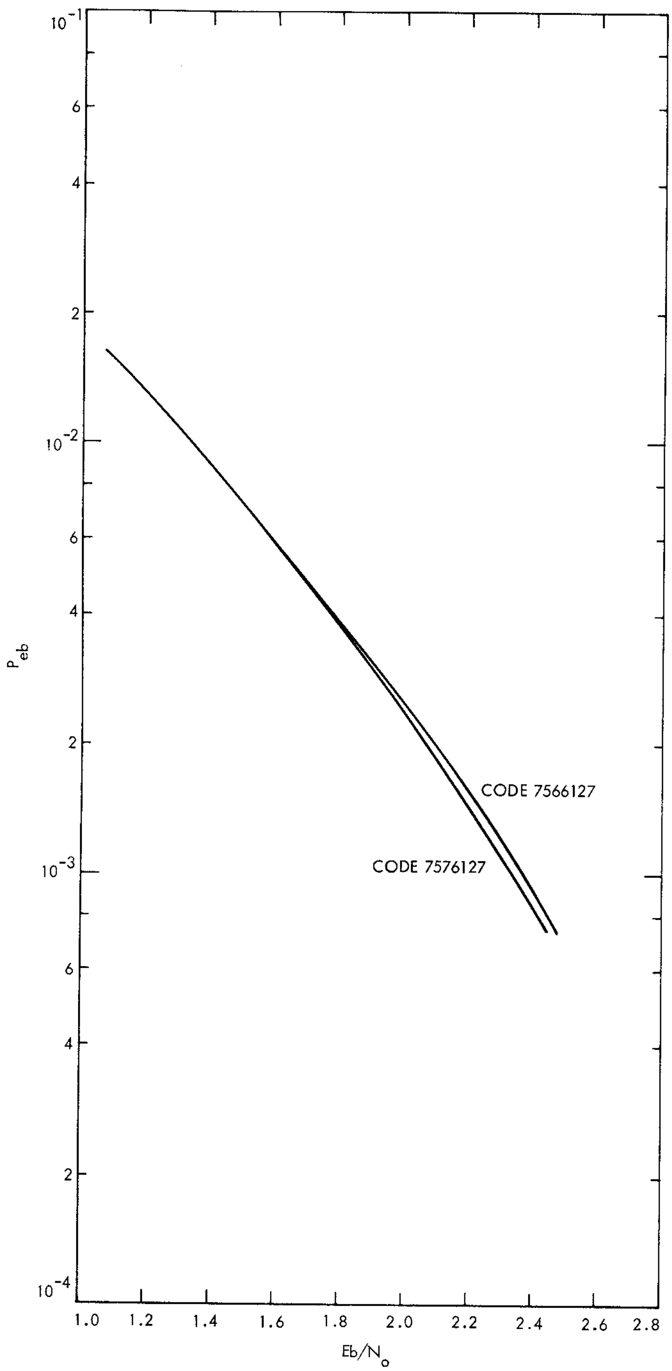

Fig. 1. Simulated bit error probability

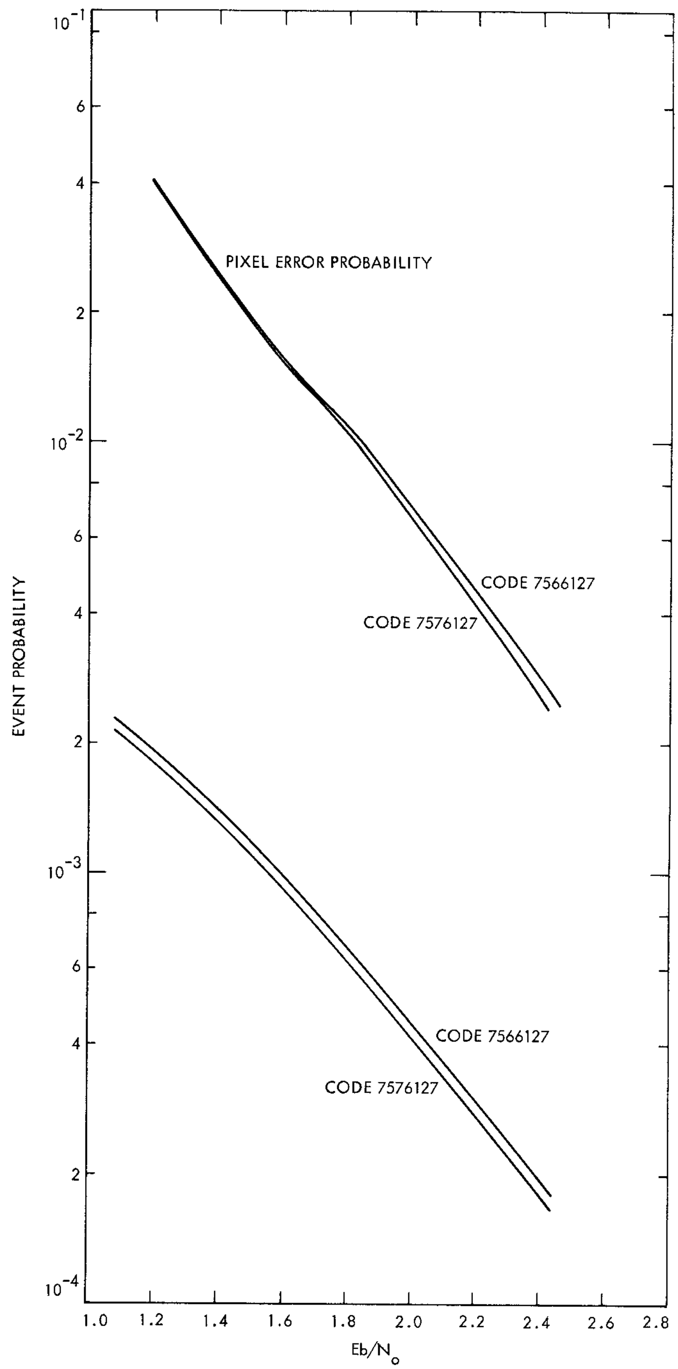

Fig. 2. Simulated pixel error probability and error event probability